Technical Comments

Comments on "Transformations for Infinite Regions and Their Applications to Flow Problems"

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THE purpose of this Comment is to further the discussion of mapping functions used to transform infinite regions into finite intervals initiated by Sills in Ref. 1. In particular, the comparison of the algebraic transformations used in this article with the velocity-related transformations used in Refs. 2 and 3 will be extended. All of these transformations provide a coordinate essentially normal to the flow direction that remains finite even when the actual physical coordinate approaches infinity. When this is achieved, highly successful finite-difference techniques implicit in the normal coordinate can be employed to solve various viscous flow problems numerically. The velocity-related transformations were derived from some bounded property of the momentum equation. For example, the Crocco transformation to velocity as the new independent variable is useful for studying bounded monotonic velocity profiles.[2] Similarly, transformation to a momentum coordinate is useful for studying jet flows.[3]

A particular algebraic transformation can be applied to a greater number of different fluid dynamic problems than a particular velocity-related transformation because it is independent of the velocity characteristics of any particular flowfield. For example, the Crocco transformation is useful for studying bounded monotonic velocity profiles only, whereas the analogous algebraic transformation of Ref. 1 is applicable regardless of the nature of the resulting velocity profile. However, a disadvantage which results from this independence is that as the solution proceeds downstream and the viscous layer grows laterally, it tends to "outgrow" the finite-difference grid of the algebraic transformation, which remains essentially fixed in space. That is, increasingly more of the flowfield is contained within the grid point corresponding to an infinite physical coordinate and the grid point immediately preceding that one. For this reason, the grid must be periodically revised as the solution continues downstream in order to properly describe the flowfield. If, however, the transformed independent variable is some bounded property of the flow, then the finite-difference grid grows "automatically," with the flowfield, and artificial downstream revisions are unnecessary. The numerical solutions of Refs. 2 and 3 enjoyed this advantage.

The velocity-related transformations lose effectiveness, however, if solutions of the energy or species equations are required in addition to the momentum equation, and the Prandtl or Schmidt numbers, respectively, are less than unity. For these cases, the enthalpy or concentration profiles extend further into the infinite region than the velocity profile. As a result, the derivatives of these quantities with respect to the velocity-related independent variable are unbounded at the transformed infinite boundary, and the velocity-related transformation fails there. Flowfield solutions using the algebraic transformations of Ref. 1 do not experience this difficulty.

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Comment on "Effects of a Dynamic Gas on Breakdown Potential"

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IN Ref. 1, the author describes an experiment in which a pair of metal electrodes were attached to the downstream end of a converging, plastic nozzle. Nonionized argon flowed transverse to the electric field at the electrodes. The argon discharged as a freejet into a vacuum chamber at the downstream end of the electrode pair. The author reports that, for fixed gas pressure in the interelectrode region, the presence of a gas flow produces a sharp reduction in the breakdown voltage of the argon below the static value. He concludes that the convective effect of a transverse gas flow *reduces* the gas breakdown potential. This conclusion is generally incorrect. The critical electric field E^* for breakdown of a nonionized gas in the presence of a transverse gas flow of velocity V is given by Eq. (10) of Ref. 2 as follows:

$$\alpha\mu_e E^* = V^2/4D_e + V_e^2/4D_e \qquad (1)$$

where α is the first Townsend ionization coefficient; and μ_e D_e and V_e are the electron mobility, diffusion coefficient, and drift velocity, respectively. An additional term involving D_e which appears on the right-hand side of Eq. (1) has been neglected because it is generally small.[2] Equation (1) shows that a transverse gas flow *increases* the breakdown electric field. For the experimental conditions of Ref. 1, $V \approx 10^4$ cm/sec and $V_e > 10^6$ cm/sec.[3] Thus, the gas flow should have a negligible effect on the breakdown process. A probable explanation for the observed reduction in breakdown

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potential for nonzero V can be deduced from Fig. 3 of Ref. 1. This figure shows that the discharge is located downstream of the electrodes, in the freejet region. In the freejet region the pressure is lower than in the interelectrode region. Consequently, the breakdown voltage in this region is lower.

In general, prior to breakdown in noble gases, the electron drift velocity term $V_e/4D_e$ in Eq. (1) is much greater than the gas velocity term, $V^2/4D_e$.[2] Therefore, the breakdown electric field of gases in subsonic or supersonic MHD devices will generally be independent of gas convective effects.

Subsequent to breakdown, one obtains[2] an expression similar to Eq. (1) for the magnitude of the electric field necessary to sustain the discharge in the presence of transverse convective effects. However, the electron drift velocity term does not appear in the equation for the discharge-sustaining voltage. One, therefore, finds that for MHD devices the gas velocity term is generally much larger than the terms due to plasma transport phenomena. Thus, the discharge is generally controlled by gas convective effects. One usually obtains a discharge structure similar to that shown in Fig. 3, Ref. 1. Further details on this phenomena can be found in Ref. 4.

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Reply by Author to B. Zauderer

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THE explanation given by Zauderer to the reduction in the breakdown potential for a dynamic condition is certainly one possibility for describing such an effect. However, the work reported in Ref. 1 indicated that the onset of breakdown does not occur in the "U-shaped" form as shown in Fig. 3 of Ref. 2, but the onset of breakdown most likely occurred within the upstream quarter of the electrode surface. The photograph shown in Fig. 4 of Ref. 1 exhibits a vertical plasma column between and within the electrode surfaces. It should be noted that the discharge shown in Fig. 3 of Ref. 2 refers to a typical *continuously* operating discharge between flat parallel-plate electrodes with the presence of a gas flow and therefore does not necessarily represent the conditions present during the onset of breakdown.

The pressure differential between the electrode region and the freejet region was measured and found to be negligible. The uniform pressure is a consequence of the subsonic velocity and the fact that the region between the electrodes was open to the vacuum chamber on all sides. Therefore this would certainly not be compatible with the measured reduction in breakdown potential of 50–80% below that required for stagnation conditions.

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Comment on "Optimum Discrete Approximation of the Maxwell Distribution"

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IN Ref. 1, a set of discrete velocities and weighting coefficients is presented which optimizes a discrete approximation to the Maxwellian equilibrium distribution function. We would like to point out that this optimization has been obtained and reported some time ago and has been utilized in the solution of several rarefied gasdynanic flow problems. In Refs. 2–4, this was referred to as the "modified quadrature" or the "half-range quadrature." The same one-dimensional values for the discrete velocities and weighting coefficients which appear in Ref. 1 also appear in Ref. 5, which due to unfortunate delays was not printed until recently. However, the values were published much earlier in Ref. 6 for the same number of discretizations (up to 8 levels).

The derivation used in Ref. 1 is essentially that of Refs. 5 and 6 although the descriptive phrasing is somewhat different. We took the approach that

$$\int_0^\infty g(v_x)e^{-v_x^2}\,dv_x \cong \sum_{j=1}^{N} H_j\, g(\alpha_j) \tag{1}$$

where H_j are the weighting coefficients corresponding to the discrete velocities α_j and $g(v_x)$ is any function for which the integral is defined. The H_j and α_j are determined under the requirement that Eq. (1) is exact if $g(v_x)$ is a polynomial of degree $2N - 1$ or less. This corresponds to duplicating the first $2N$ moments of a normalized Maxwellian distribution. If g is not such a polynomial, then an error is incurred which depends upon the departure of g from this form. Equation (1) leads to the equation

$$\sum_{j=1}^{N} H_j\alpha_j{}^k = \int_0^\infty v_x{}^k e^{-v_x^2}\,dv_x,\; k = 0,1,\ldots,2N-1 \tag{2}$$

which is to be compared to Eq. (7) of Ref. 1

$$\sum_{i=1}^{N} K_i v_i{}^j = \frac{1}{(\pi)^{1/2}}\int_0^\infty v^j e^{-v^2}\,dv,\; j = 0,1,\ldots,2N-1$$

Thus, the one-dimensional K_i of Ref. 1 and the H_j of Ref. 5 differ by a factor of $1/(\pi)^{1/2}$ due to the normalizations used. The α_j and v_i are identical.

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